# Pre: Multinomial Coefficients

Math model: To count the number of ways to partition a finite set into disjoint subsets. This generalizes binomial coefficients.

## Example: Choosing Committees.

Suppose there are 20 members of an organization are to be divided into 3 committees A, B and C in such a way that each of A and B is to have 8 members and C is to have 4 members. We shall determine the number of ways in which members can be assigned to these committees. Notice that each of the 20 members gets assigned to one and only one committee.

**Answer:**

One way to think of the assignments is to form committee A first by choosing 8 members, and then split the remaining 12 members into committee B and C. Each of these operation is a combination problem.

Note that how the 12! that appears in the denominator of divides out with the 12! that appears in the numerator of .

## Generalize of the example

Suppose that distinct elements are to be divided into different groups in such a way that:

for , the th group contains exactly elements, where .

It is desired to determine the number of different ways in which elements can be divided into groups.

The elements in the first group can be selected from the n available elements in different ways, and the elements in the second group can be selected from the remaining elements in different ways, and etc. Hence “the total number of different ways of selecting the elements for the first groups” is

Note that after selecting the elements for the first k-1 groups, the elements for the kth group is automatically selected. Thus the total number of ways is

## Multinomial Coefficients & Multinomial Theorem

**Multinomial coefficients:**

**Multinomial theorem:**

where the summation extends over all possible combinations of nonnegative integers such that .

# Multinomial Distribution

## Model & derivation

Suppose:

* A population(总体) contains different types of items.
* The proportion of the items in the population that are of type is , where  
   for , and .
* items are selected at random from the population, with replacement(有放回).

Let denote the vector of the probabilities of each type. Let denote the number of selected items that are of type .

Because the items are selected at random with replacement, the selections will be **independent** of each other. Hence, the probability that “the first selection is of type , the second selection is of type , …, and the th selection is of type ” is simply

Suppose such sequence consists of exactly items of type 1, items of type 2, and so on. Then,

It follows that “obtaining exactly items of type ” has ways. Thus the probability of event is

## Notation

Let denote the random vector of counts for types,   
let denote the vector of the probabilities of each type,   
and let denote a possible value for that vector.   
Finally, let denote the joint p.f of . Then

## Relation with Binomial Distributions

**Corollary: Marginal Distribution of MD**

Suppose that random vector has the multinomial distribution with parameters and . The marginal distribution of each variable is the binomial distribution with parameters n and .

**Corollary: Marginal Distribution of the partial sum of coordinates**

The marginal distribution of the sum of some of the coordinates of a multinomial vector has a binomial distribution.

**Theorem: Means, Variances and Covariances**

**Proof**

From the marginal distribution of MD, the first two formula is obvious.

From the marginal distribution of the partial sum of the coordinates, has the binomial distribution with parameters and . Hence,

According to the theorem of **“the variance of the sum of random variables”** (Morris. DeGroot. Probability and Statistics, 4th edition. Theorem 4.6.6),

Thus can be solved.